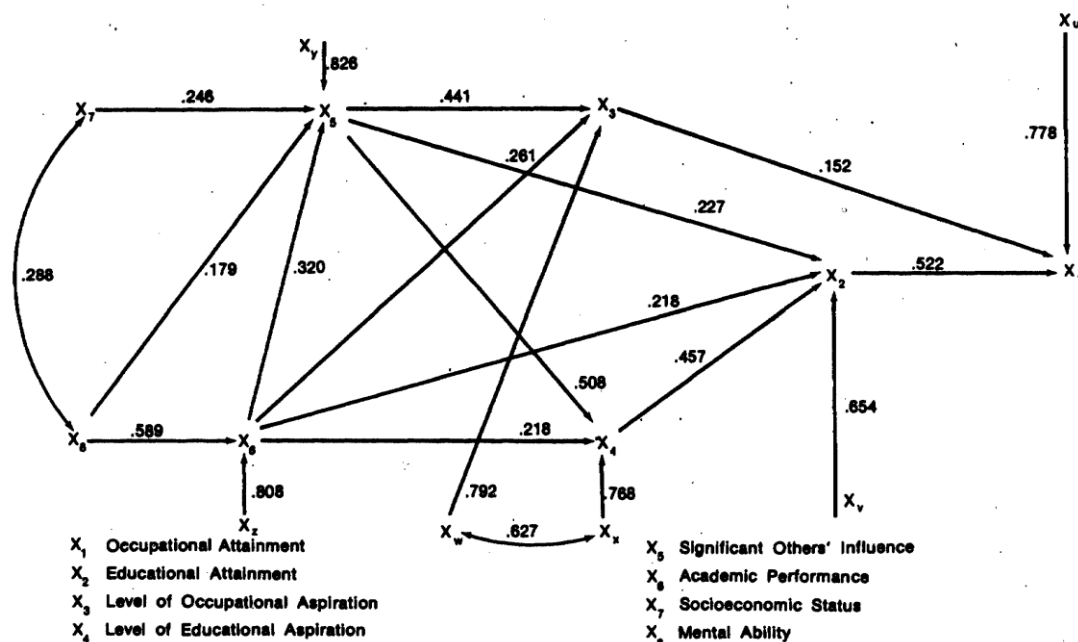


## Intro to path analysis

Sources. This discussion draws heavily from Otis Dudley Duncan's Introduction to Structural Equation Models.

Overview. Our theories often lead us to be interested in how a series of variables are interrelated. It is therefore often desirable to develop a system of equations, i.e. a model, which specifies all the causal linkages between variables. For example, status attainment research asks how family background, educational attainment and other variables produce socio-economic status in later life. Here is one of the early status attainment models (see Hauser, Tsai, Sewell 1983 for a discussion):



**Figure 2. Sewell-Haller-Ohlendorf Model of Educational and Occupational Attainment**  
 Source: William H. Sewell, Archibald O. Haller and George W. Ohlendorf, *The Educational and Early Occupational Status Attainment Process: Replication and Revision*. *American Sociological Review* 35 (December 1970): 1023.

Among the many implications of this model are that Parents' Socio-Economic Status ( $X_7$ ) indirectly affects the Educational Attainment ( $X_2$ ) and Occupational Aspirations ( $X_3$ ) of children. These, in turn, directly affect children's Occupational Attainment ( $X_1$ ). In other words, higher parental SES helps children to become better educated and gives them higher occupational aspirations, which in turn leads to greater occupational achievement. Our earlier discussion of the Logic of Causal Order, combined with the current discussion of Path Analysis, can help us better understand how models such as the above work.

Review of key lessons from the logic of causal order. In the logic of causal order, we learned that the correlation between two variables says little about the causal relationship between them. This is because the correlation between two variables can be due to

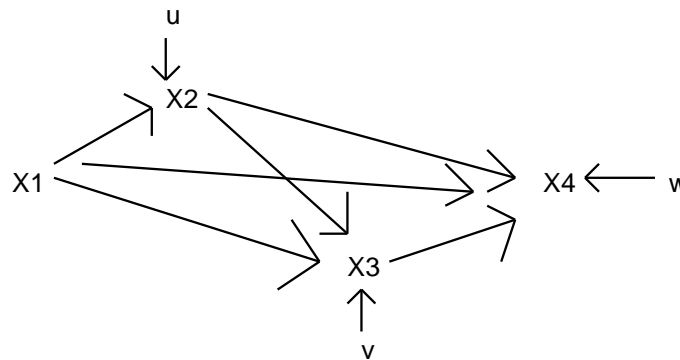
- the direct effect of one variable on another
- indirect effects; one variable affects another variable which in turn affects a third
- common causes, e.g. X affects both Y and Z. This is spurious association
- correlated causes, e.g. X is a cause of Z and X is correlated with Y
- reciprocal causation; each variable is a cause of the other

Hence, a correlation can reflect many non-causal influences. Further, a correlation can't tell you anything about the direction of causality.

At the same time, only looking at the direct effect of one variable on another may also not be optimal. Direct effects tell you how a 1 unit change in X will affect Y, holding all other variables constant. However, it may be that other variables are not likely to remain constant if X changes, e.g. a change in X can produce a change in Z which in turn produces a change in Y. Put another way, both the direct and indirect effects of X on Y must be considered if we want to know what effect a change in X will have on Y, i.e. we want to know the *total effects* (direct + indirect).

We have done all this conceptually. Now, we will see how, using path analysis, this is done mathematically and statistically. We will show how the correlation between two variables can be *decomposed* into its component parts, i.e. we will show how much of a correlation is due to direct effects, indirect effects, common causes and correlated causes. We will further show how each of the *structural effects* in a model affects the correlations in the model.

Path analysis terminology. Consider the following diagram:



In this diagram,

- X1 is an *exogenous variable*. Exogenous variables are those variables whose causes are not explicitly represented in the model. Exogenous variables are causally prior to all dependent variables in the model. There is no causal ordering of the exogenous variables. There can be more than one exogenous variable in a model. For example, if there was a 2-headed arrow linking X1 and X2 instead of a 1-headed arrow, then X1 and X2 would both be exogenous.
- Conversely, X2, X3, and X4 are *endogenous variables*. The causes of endogenous variables are specified in the model.
- Exogenous variables must always be independent variables. However, endogenous variables can be either dependent or independent. For example, X1 is a cause of X2, but X2 is itself a cause of X3 and X4.

- $u$ ,  $v$ , and  $w$  are *disturbances*, or, if you prefer, the residual terms. Many notations are used for disturbances; indeed, sometimes no notation is used at all, there is just an arrow coming in from out of nowhere.  $\varepsilon_2$ ,  $\varepsilon_3$ , and  $\varepsilon_4$  would also be a good notation, given our past practices.
- The one way arrows represent the direct causal effects in the model, also known as the *structural effects*. Sometimes, the names for these effects are specifically labeled, but other times they are left implicit. The *structural equations* in the above diagram can be written as

$$X_2 = \beta_{21}X_1 + u$$

$$X_3 = \beta_{31}X_1 + \beta_{32}X_2 + v$$

$$X_4 = \beta_{41}X_1 + \beta_{42}X_2 + \beta_{43}X_3 + w$$

- Note that we use 2 subscripts for each structural effect. The first subscript stands for the DV, the second stands for the IV. When there are multiple equations, this kind of notation is necessary to keep things straight. Note, too, that intercepts are not included. Discussions of path analysis are simplified by assuming that all variables are “centered,” i.e. the mean of the variable has been subtracted from each case. Finally, note that the paths linking the disturbances to their respective variables are set equal to 1.
- In the above example, each IV was affected by all the other *predetermined variables*, i.e. those variables which are causally prior to it. We refer to such a model as being *fully recursive*, for reasons we will explain later. There is no requirement that each IV be affected by all the predetermined variables, of course. For example,  $\beta_{43}$  could equal zero, in which case that path would be deleted from the model. Indeed, it is fairly easy to include paths in a model; the theoretically difficult part is deciding which paths to leave out.

Determining correlations and coefficients in a path model using standardized variables. We will now start to examine the mathematics behind a path model. For convenience, WE WILL ASSUME THAT ALL VARIABLES HAVE A MEAN OF 0 AND A VARIANCE OF 1, i.e. are standardized. This makes the math easier, and it is easy enough later on to go back to unstandardized variables. Recall that, when variables are standardized,

$$E(X_1^2) = V(X_1) = 1,$$

$$E(X_1X_2) = \text{COV}(X_1, X_2) = \rho_{12} \text{ (where } \rho_{12} \text{ is the population counterpart to the sample estimate } r_{12}\text{)}$$

Also, we assume (at least for now) that the disturbance in an equation is uncorrelated with any of the IVs in the equation. (Note, however, that the disturbance in each equation has a nonzero correlation with the dependent variable in that equation and (in general) with the dependent variable in each “later” equation.)

Keeping the above in mind, if we know the structural parameters, it is fairly easy to compute the underlying correlations. Perhaps more importantly, it is possible to decompose the correlation between two variables into the sources of association noted above, e.g. correlation due to direct effects, correlation due to indirect effects, etc. And, of course, if we know the correlations, we can compute the structural parameters, although this is somewhat harder to do by hand.

There are a couple of ways of doing this. The first, Sewell Wright's rule, is very diagram-oriented and is perhaps more intuitive to most people once you understand it. The normal equations approach is more mathematical; while perhaps less intuitive, it is less prone to mistakes. I find that using both together is often helpful.

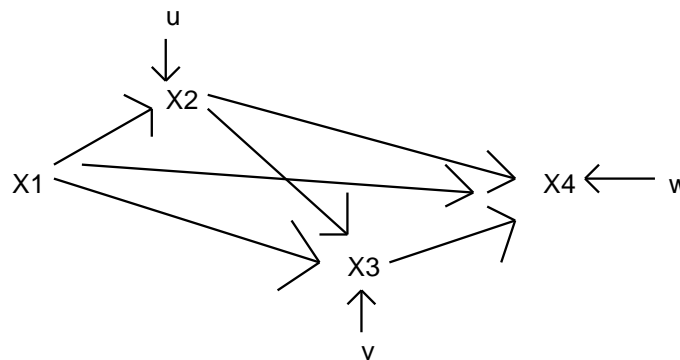
**Sewell Wright's multiplication rule:** To find the correlation between  $X_h$  and  $X_j$ , where  $X_j$  appears "later" in the model,

- begin at  $X_j$  and read *back* to  $X_h$  along each distinct direct and indirect (compound) path, forming the product of the coefficients along that path. (This will give you the correlation between  $X_j$  and  $X_h$  that is due to the direct and indirect effects of  $X_h$  on  $X_j$ )
- After reading back, read *forward* (if necessary), but only one reversal from back to forward is permitted. (This will give you correlation that is due to common causes.)
- A double-headed arrow may be read either forward or backward, but you can only pass through 1 double-headed arrow on each transit. (This will give you correlation due to correlated causes)
- If you pass through a variable, you may not return to it on that transit.
- Sum the products obtained for all the linkages between  $X_j$  and  $X_h$ . (The main trick to using Wright's rule is to make sure you don't miss any linkages, count linkages twice, or make illegal double reversals.) This will give you the total correlation between the 2 variables.

**Normal equations.** To get the normal equations, each structural equation is multiplied by its predetermined variables, and then expectations are taken. If the structural parameters are known, simple algebra then yields the correlations. We'll show how to use normal equations in the more complicated example.

To illustrate path analysis principles, we'll first go over a generic and complicated example. We'll then present a fairly simple substantive (albeit hypothetical) example similar to what we've discussed before.

**Generic, Complicated Example** (pretty much stolen from Duncan). We will illustrate both the Wright rule and the use of normal equations for each of the 3 structural equations in the model presented earlier:



(1) X2. For X2, the structural equation is

$$X_2 = \beta_{21}X_1 + u$$

The only predetermined variable is X1. Hence, if we multiply both sides of the above equation by X1 and then take expectations, we get the normal equation

$$E(X_1X_2) = \beta_{21}E(X_1^2) + E(X_1u) =$$

$$\rho_{21} = \beta_{21}$$

NOTE: How did we get from the structural equation to the normal equation? First, we multiplied both sides of the structural equation by X1, and then we took the expectations of both sides, i.e.

$$X_2 = \beta_{21}X_1 + u \Rightarrow$$

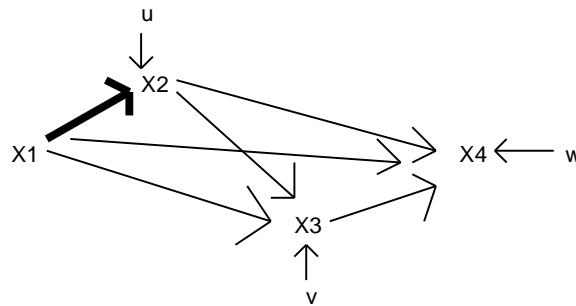
$$X_1X_2 = \beta_{21}X_1^2 + X_1u \Rightarrow$$

$$E(X_1X_2) = \beta_{21}E(X_1^2) + E(X_1u) =$$

$$\rho_{21} = \beta_{21}$$

Again, remember that when variables are standardized,  $E(X_1^2) = 1$  and  $E(X_1X_2) = \rho_{12}$  (where  $\rho_{12}$  is the population counterpart to the sample estimate  $r_{12}$ ). Also remember that we are assuming that the disturbance in an equation is uncorrelated with any of the IVs in the equation, ergo  $E(X_1u) = 0$ .

Hence, as we have seen before, in a bivariate regression, the correlation is the same as the standardized regression coefficient. Also, all of the correlation between X1 and X2 is causal.



SW Rule: Go back from X2 to X1.

(2) X3. For X3, the structural equation is

$$X_3 = \beta_{31}X_1 + \beta_{32}X_2 + v$$

There are two predetermined variables, X1 and X2. Taking each in turn, the normal equations are

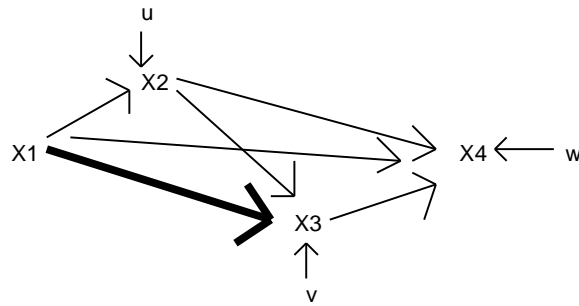
$$E(X_1X_3) = \beta_{31}E(X_1^2) + \beta_{32}E(X_1X_2) + E(X_1v) =$$

$$\rho_{13} = \beta_{31} + \beta_{32}\rho_{12}$$

$$= \beta_{31} + \beta_{32}\beta_{21}$$

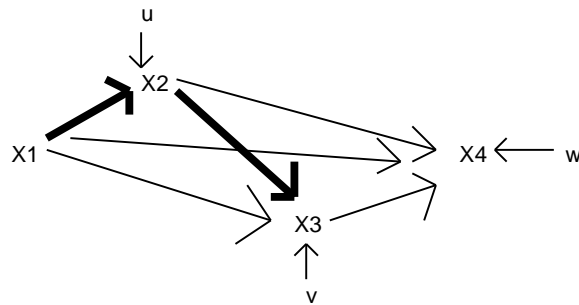
(Remember that  $\beta_{21} = \rho_{12}$ ). As the above makes clear, there are two sources of correlation between X1 and X3:

- (a) There is a direct effect of X1 on X3 (represented in  $\beta_{31}$ )



SW Rule: Go back from X3 to X1.

- (b) An indirect effect of X1 operating through X2 (reflected by  $\beta_{32}\beta_{21}$ ). All of the association between X1 and X3 is causal.



SW Rule: Go back from X3 to X2, and then back from X2 to X1.

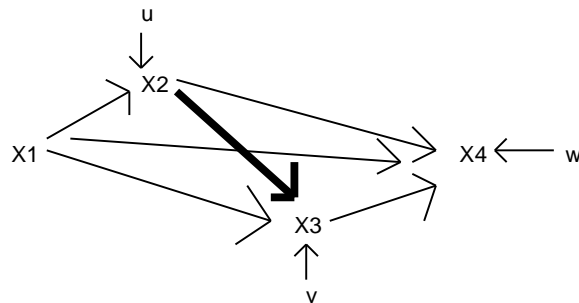
NOTE: Recall that the sum of a variable's direct effect and its indirect effects is known as its *total effect*. So, in this case, the total effect of X1 on X3 is  $\beta_{31} + \beta_{32}\beta_{21}$ .

Doing the same thing for X2 and X3, we get

$$\begin{aligned}
 E(X_2X_3) &= \beta_{31}E(X_1X_2) + \beta_{32}E(X_2^2) + E(X_2v) = \\
 \rho_{23} &= \beta_{31}\rho_{12} + \beta_{32} \\
 &= \beta_{31}\beta_{21} + \beta_{32}
 \end{aligned}$$

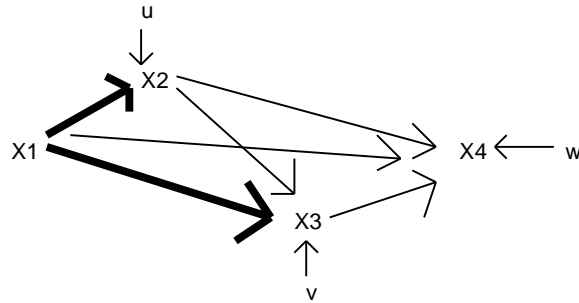
Again, as the above makes clear, there are two sources of correlation between X2 and X3:

- (a) There is a direct effect of X2 on X3 (represented in  $\beta_{32}$ ).



SW Rule: Go back from X3 to X2.

(b) But, there is also correlation due to a common cause, X1 (reflected by  $\beta_{31}\beta_{21}$ ). Hence, part of the correlation between X2 and X3 is spurious.



SW Rule: Go back from X3 to X1, go forward from X1 to X2.

(3) X4. For X4, the predetermined variables are X1, X2, and X3. The structural equation is

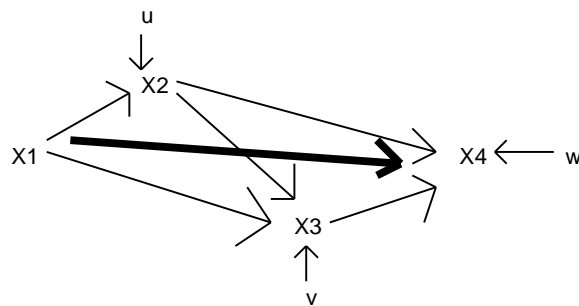
$$X_4 = \beta_{41}X_1 + \beta_{42}X_2 + \beta_{43}X_3 + w$$

The normal equations are, first, for X1,

$$\begin{aligned} E(X_1X_4) &= \beta_{41}E(X_1^2) + \beta_{42}E(X_1X_2) + \beta_{43}E(X_1X_3) + E(X_1w) = \\ \rho_{41} &= \beta_{41} + \beta_{42}\rho_{12} + \beta_{43}\rho_{13} \\ &= \beta_{41} + \beta_{42}\beta_{21} + \beta_{43}(\beta_{31} + \beta_{32}\beta_{21}) \\ &= \beta_{41} + \beta_{42}\beta_{21} + \beta_{43}\beta_{31} + \beta_{43}\beta_{32}\beta_{21} \end{aligned}$$

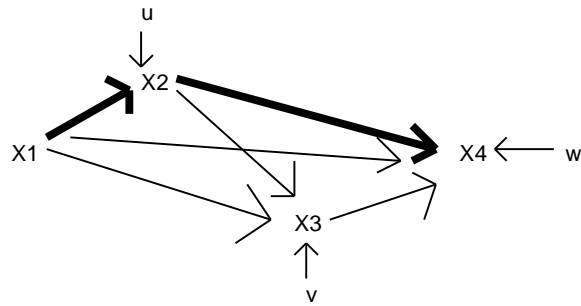
This shows there are 4 sources of association between X1 and X4:

(a) Association due to the direct effect of X1 on X4 ( $\beta_{41}$ )



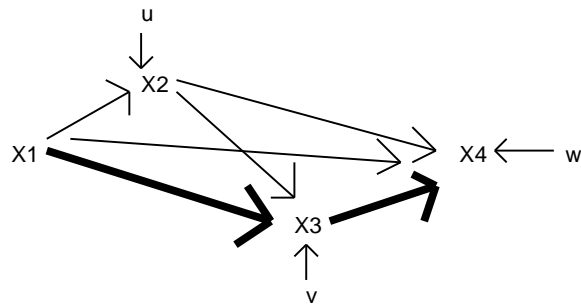
SW Rule: Go back from X4 to X1.

(b) Association due to an indirect effect: X1 affects X2 which then affects X4 ( $\beta_{42}\beta_{21}$ )



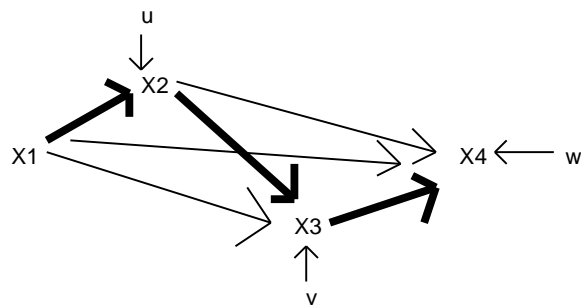
SW Rule: Go back from X4 to X2, go back from X2 to X1.

(c) Association due to another indirect effect: X1 affects X3 which then affects X4 ( $\beta_{43}\beta_{31}$ )



SW Rule: Go back from X4 to X3, go back from X3 to X1.

(d) Association due to yet another indirect effect: X1 affects X2, which then affects X3, which then affects X4 ( $\beta_{43}\beta_{32}\beta_{21}$ )



SW Rule: Go back from X4 to X3, back from X3 to X2, back from X2 to X1.

Note that you sum (b), (c) and (d) to get the total indirect effect of X1 on X4. Note too that all of the correlation between X1 and X4 is causal.

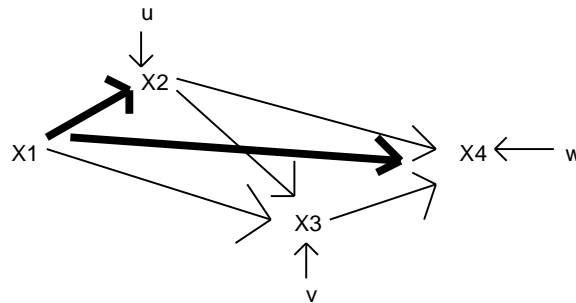


The normal equations for X2 and X4 are

$$\begin{aligned}
 E(X_2 X_4) &= \beta_{41} E(X_2 X_1) + \beta_{42} E(X_2^2) + \beta_{43} E(X_2 X_3) + E(X_2 w) = \\
 \rho_{42} &= \beta_{41} \rho_{12} + \beta_{42} + \beta_{43} \rho_{23} \\
 &= \beta_{41} \beta_{21} + \beta_{42} + \beta_{43} (\beta_{32} + \beta_{31} \beta_{21}) \\
 &= \beta_{41} \beta_{21} + \beta_{42} + \beta_{43} \beta_{32} + \beta_{43} \beta_{31} \beta_{21}
 \end{aligned}$$

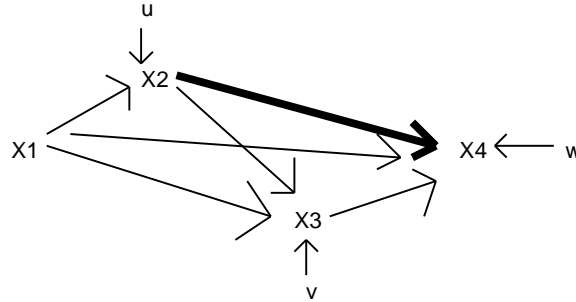
This shows there are 4 sources of association between X2 and X4:

(a) Association due to X1 being a common cause of X2 and X4 ( $\beta_{41}\beta_{21}$ )



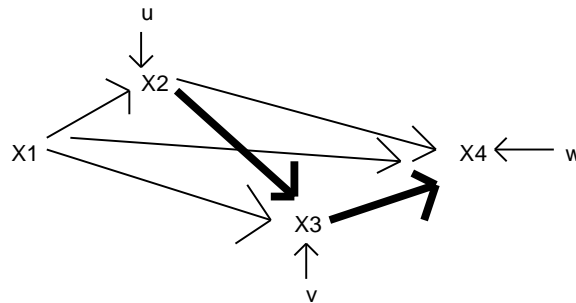
SW Rule: GO back from X4 to X1, go forward from X1 to X2.

(b) Association due to the direct effect of X2 on X4 ( $\beta_{42}$ )



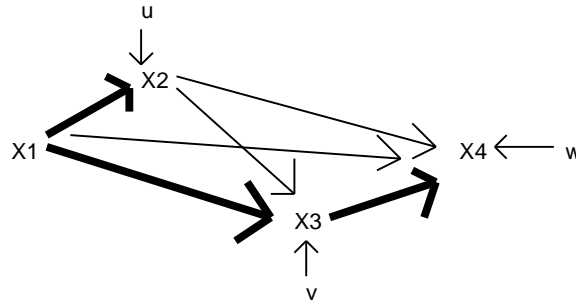
SW Rule: Go back from X4 to X2.

(c) Association due to the indirect effect of X2 affecting X3 which in turn affects X4 ( $\beta_{43}\beta_{32}$ )



SW Rule: Go back from X4 to X3, go back from X3 to X2.

(d) Association due to X1 being a common cause of X2 and X4: X1 directly affects X2 and indirectly affects X4 through X3 ( $\beta_{43}\beta_{31}\beta_{21}$ ).



SW Rule: Go back from X4 to X3, back from X3 to X1, forward from X1 to X2.

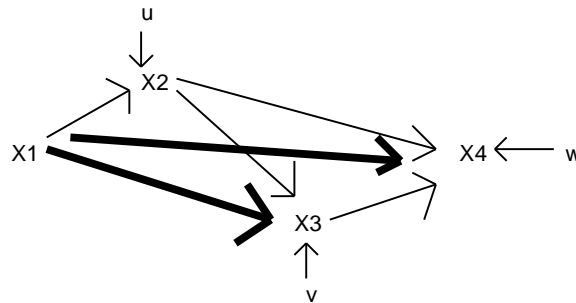
Note that you sum (a) and (d) to get the correlation due to common causes. This represents spurious association, while (b) + (c) represents causal association.

The normal equations for X3 and X4 are,

$$\begin{aligned}
 E(X_3X_4) &= \beta_{41}E(X_3X_1) + \beta_{42}E(X_3X_2) + \beta_{43}E(X_3^2) + E(X_3w) = \\
 \rho_{43} &= \beta_{41}\rho_{13} + \beta_{42}\rho_{23} + \beta_{43} \\
 &= \beta_{41}(\beta_{31} + \beta_{32}\beta_{21}) + \beta_{42}(\beta_{32} + \beta_{31}\beta_{21}) + \beta_{43} \\
 &= \beta_{41}\beta_{31} + \beta_{41}\beta_{32}\beta_{21} + \beta_{42}\beta_{32} + \beta_{42}\beta_{31}\beta_{21} + \beta_{43}
 \end{aligned}$$

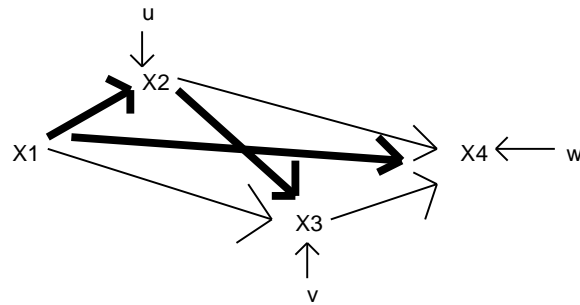
This shows there are 5 sources of association between X3 and X4:

(a) Association due to X1 being a common cause of X3 and X4 ( $\beta_{41}\beta_{31}$ )



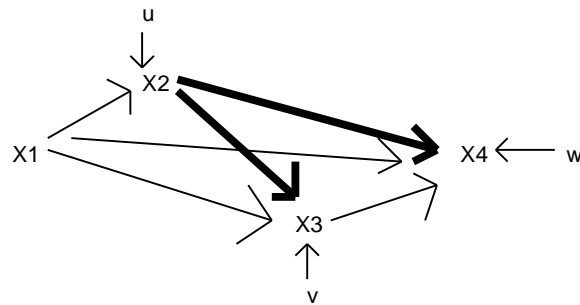
SW Rule: Go back from X4 to X1, go forward from X1 to X3.

(b) Association due to X1 being a common cause of X3 (by first affecting X2, which in turn affects X3) and X4 ( $\beta_{41}\beta_{21}\beta_{32}$ )



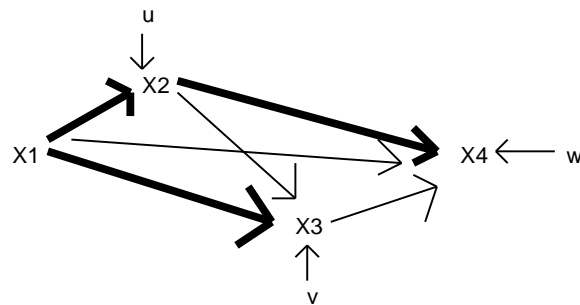
SW rule: Go back from X4 to X1, forward from X1 to X2, forward from X2 to X3.

(c) Association due to X2 being a common cause of X3 and X4 ( $\beta_{42}\beta_{32}$ )



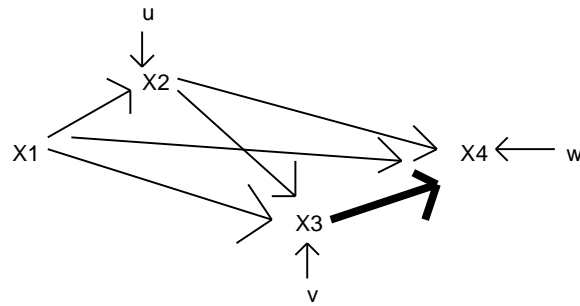
SW Rule: Back from X4 to X2, go forward from X2 to X3.

(d) Association due to X1 being a common cause of X3 and X4: X1 directly affects X3 and indirectly affects X4 through X2 ( $\beta_{42}\beta_{21}\beta_{31}$ ).



SW Rule: Go back from X4 to X2, back from X2 to X1, forward from X1 to X3.

(e) Association due to X3 being a direct cause of X4 ( $\beta_{43}$ )



SW Rule: Go back from X4 to X3.

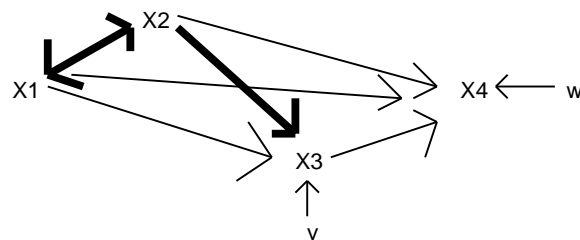
Note that you sum (a), (b) (c) and (d) to get the correlation due to common causes. This is the spurious association. There are no indirect effects of X3 on X4.

In reviewing the above, note that, *if there are no double-headed arrows in the model*

- If you go back once and then stop, it is a direct effect
- If you go back 2 or more times and never come forward, it is an indirect effect
- If you go back and later come forward, it is correlation due to a common cause

**Correlated causes.** Suppose that, in the above model, X1 and X2 were both exogenous, i.e. there was a double-headed arrow between them instead of a 1-way arrow. This would not have any significant effect on the math, but it would affect our interpretation of the sources of correlation. Anything involving  $\rho_{12}$  would then have to be interpreted as correlation due to correlated causes. Further, we could not always say what effect changes in X1 would have on other variables, since we wouldn't know whether changes in X1 would also produce changes in X2 (unless we have good reasons for believing that that couldn't be the case, e.g. gender and race might both be exogenous variables in a model, but we are pretty confident that changes in one are not going to produce changes in the other.). That is, with two-headed arrows we often can't be sure what the indirect effects are, which also means that we can't be sure what the total effects are. Ergo, *the fewer 2-headed arrows in a model, the more powerful the model is in terms of the statements it makes.*

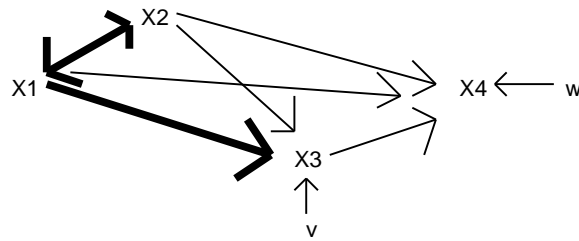
For example:



Instead of X1 and X3 being correlated because of the indirect effect of X1 affecting X2 which in turn affects X3 (which is a causal relationship) X1 and X3 are correlated because of the

correlated causes of X1 and X2 (which we do not assume to be causal), i.e. X1 is correlated with a cause of X3.

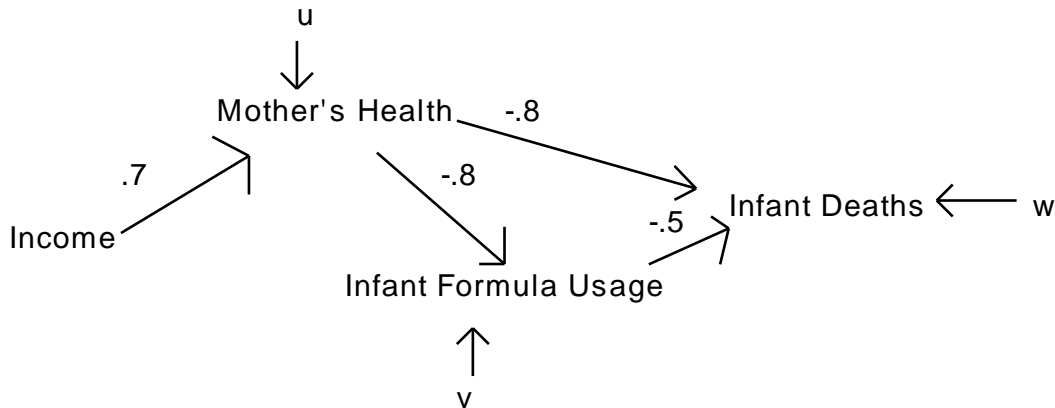
Or,



Instead of X2 and X3 being correlated because they share a common cause, they are correlated because of a correlated cause, i.e. X1 is a cause of X3 and X2 is correlated with X1.

**SUBSTANTIVE HYPOTHETICAL EXAMPLE** (Adapted From the 1995 Soc 593 Exam 2):

A demographer believes that the following model describes the relationship between Income, Health of the Mother, Use of Infant formula, and Infant deaths. All variables are in standardized form. The hypothesized value of each path is included in the diagram.



- a. Write out the structural equation for each endogenous variable.

$$MH = \beta_{MH,Inc} Income + u = .7 * Income + u$$

$$IF = \beta_{IF,MH} MH + v = -.8 * MH + v$$

$$ID = \beta_{ID,MH} * MH + \beta_{ID,IF} * IF + w = -.8 * MH - .5 * IF + w$$

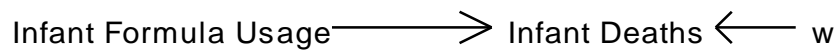
- b. Determine the complete correlation matrix. (Remember, variables are standardized. You can use either normal equations or Sewell Wright, but you might want to use both as a double-check.)

Correlation	Sewell-Wright Approach
$r_{mh,inc} = .7$	Go back from Mother's health to Income. (Direct effect of Income on MH)
$r_{if,MH} = -.8$	Go back from IF to MH. (Direct effect of MH on IF)
$r_{IF,Inc} = -.8 * .7 = -.56$	Go back from IF to MH, then back from MH to income. (Indirect effect of Income – Income affects mother's health which in turn affects Infant formula usage)
$r_{id,IF} = -.5 + -.8 * -.8 = .14$	<p>Go back from ID to IF. (Direct effect of Infant formula on infant deaths)</p> <p>Then, go back from ID to MH, then go forward from MH to IF. (Mother's health is a common cause of both Infant formula usage and infant deaths)</p> <p>Note that, even though the direct effect of infant formula usage on infant deaths is negative (which means that using formula reduces infant deaths) the correlation between infant formula usage and infant deaths is positive (which means that those who use formula are more likely to experience infant deaths). We discuss this further below.</p>
$r_{id,MH} = -.8 + -.8 * -.5 = -.4$	<p>Go back from ID to MH. (Direct effect of Mother's Health on Infant deaths)</p> <p>Then, go back from ID to IF to MH. (Indirect effect of Mother's health on infant deaths – Mother's health affects infant formula usage which in turn affects infant deaths)</p>
$r_{id,INC} = -.8 * .7 + -.5 * -.8 * .7 = -.28$	<p>Go back from Infant Death to Mother's Health, then back to Income. (Income is an indirect cause of Infant deaths – Income affects mother's health which in turn affects infant deaths.)</p> <p>Then go back from Infant deaths, then back to Mother's Health, then back to Income. (Income is yet again an indirect cause – Income affects Mother's Health, which affects Infant Formula Usage, which affects Infant Deaths.)</p>

c. Decompose the correlation between Infant deaths and Usage of Infant formula into

- Correlation due to direct effects  
-.5 (see path from IF to ID)
- Correlation due to common causes  
-.8 \* -.8 = .64 (Mother's health is a cause of both IF and ID)

d. Suppose the above model is correct, but instead the researcher believed in and estimated the following model:



What conclusions would the researcher likely draw? Why would he make these mistakes? Discuss the consequences of this mis-specification.

The correlation between IF and ID is positive, hence, if the above model was estimated, the expected value of the coefficient would be .14. This would imply that infant formula usage increases infant deaths, when in reality the correct model shows that it decreases them. The correlation is positive because of the common cause of Mother's health: less healthy mothers are more likely to use infant formula, and they are also more likely to have higher infant death rates. Belief in the above model could lead to a reduction in infant formula usage, which would have exactly the opposite effect of what was intended.

## Appendix: Verifying answers with Stata and SPSS

**Stata.** We can use Stata to verify we have calculated the correlations correctly. Just give Stata the correlations we computed by hand and then run the various regressions. If we've done everything right, the regression parameters should come out the same as in the path diagram. Remember, this is easier if you use the "input matrix by hand" submenu. (Click Data/ Matrices / Input matrix by hand.)

```
. matrix input Corr = (1, .7, -.56, -.28 \ .7, 1, -.80, -.40 \ -.56, -.80, 1, .14 \ -.28, -.40, .14, 1)
. matrix input SDs = (1,1,1,1)
. matrix input Means = (0,0,0,0)
. corr2data income mhealth formula death, corr(Corr) mean(Means) sd(SDs) n(100)
(obs 100)
```

```
. reg mhealth income
```

Source	SS	df	MS			
Model	48.5099991	1	48.5099991	Number of obs =	100	
Residual	50.4899995	98	.515204077	F( 1, 98) =	94.16	
Total	98.9999987	99	.999999986	Prob > F =	0.0000	
				R-squared =	0.4900	
				Adj R-squared =	0.4848	
				Root MSE =	.71778	

mhealth	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
income	.7	.0721393	9.70	0.000	.5568419	.8431581
_cons	6.41e-10	.0717777	0.00	1.000	-.1424405	.1424405

```
. reg formula income mhealth
```

Source	SS	df	MS			
Model	63.3600001	2	31.68	Number of obs =	100	
Residual	35.64	97	.367422681	F( 2, 97) =	86.22	
Total	99.0000001	99	1	Prob > F =	0.0000	
				R-squared =	0.6400	
				Adj R-squared =	0.6326	
				Root MSE =	.60615	

formula	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
income	4.93e-09	.0853061	0.00	1.000	-.1693091	.1693091
mhealth	-.8	.0853061	-9.38	0.000	-.9693091	-.6306909
_cons	-2.31e-09	.0606154	-0.00	1.000	-.1203048	.1203048



```
. reg death income mhealth formula
```

Source	SS	df	MS			
Model	24.749999	3	8.24999966	Number of obs =	100	
Residual	74.2500011	96	.773437511	F( 3, 96) =	10.67	
Total	99.0000001	99	1	Prob > F =	0.0000	
				R-squared =	0.2500	
				Adj R-squared =	0.2266	
				Root MSE =	.87945	

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
death						
income	1.63e-09	.1237684	0.00	1.000	-.2456784	.2456784
mhealth	-.8	.1709021	-4.68	0.000	-1.139238	-.4607621
formula	-.5	.1473139	-3.39	0.001	-.7924158	-.2075842
_cons	-6.54e-09	.0879453	-0.00	1.000	-.17457	.17457

```
. * The mis-specified model
. reg death formula
```

Source	SS	df	MS			
Model	1.94039993	1	1.94039993	Number of obs =	100	
Residual	97.0596001	98	.990404083	F( 1, 98) =	1.96	
Total	99.0000001	99	1	Prob > F =	0.1648	
				R-squared =	0.0196	
				Adj R-squared =	0.0096	
				Root MSE =	.99519	

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
death						
formula	.14	.1000204	1.40	0.165	-.0584872	.3384872
_cons	-5.23e-09	.099519	-0.00	1.000	-.1974923	.1974923

SPSS. We can also use SPSS to verify we have calculated the correlations correctly.

```
MATRIX DATA VARIABLES = Income MHealth Formula Death/ FORMAT = FREE lower
                        /FILE = INLINE / N = 100
                        /CONTENTS = CORR Mean Stddev.
BEGIN DATA.
  1.00
  .70    1.00
  -.56   -.80    1.00
  -.28   -.40    .14    1.00
  0       0       0       0
  1       1       1       1
END DATA.

* Mhealth Dependent.
REGRESSION  matrix = in(*)
            /VARIABLES Income MHealth Formula Death
            /dependent MHealth
            /method enter Income.
```

## Regression

Coefficients<sup>a</sup>

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.000	.072		.000	1.000
	INCOME	.700	.072	.700	9.703	.000

a. Dependent Variable: MHEALTH

\* Formula dependent.

```
REGRESSION  matrix = in(*)
           /VARIABLES Income MHealth Formula Death
           /dependent Formula
           /method enter Income MHealth.
```

## Regression

**Coefficients<sup>a</sup>**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.000	.061		.000	1.000
	INCOME	.000	.085	.000	.000	1.000
	MHEALTH	-.800	.085	-.800	-9.378	.000

a. Dependent Variable: FORMULA

\* Death dependent.

```
REGRESSION  matrix = in(*)
           /VARIABLES Income MHealth Formula Death
           /dependent Death
           /method enter Income MHealth Formula.
```

## Regression

**Coefficients<sup>a</sup>**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.000	.088		.000	1.000
	INCOME	.000	.124	.000	.000	1.000
	MHEALTH	-.800	.171	-.800	-4.681	.000
	FORMULA	-.500	.147	-.500	-3.394	.001

a. Dependent Variable: DEATH

\* Mis-specified Death model -- Mother's Health omitted.

```
REGRESSION  matrix = in(*)
           /VARIABLES Income MHealth Formula Death
           /dependent Death
           /method enter Formula.
```

## Regression

**Coefficients<sup>a</sup>**

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.000	.100		.000	1.000
	FORMULA	.140	.100	.140	1.400	.165

a. Dependent Variable: DEATH

<http://www.nd.edu/~rwilliam/xsoc63993/162.pdf>